Therefore, in conclusion it can be emphasized that a study of the instantaneous temperature field displayed the significance of temperature fluctuations in the separation domain, especially for low degrees of stream compression, which is not always detected successfully in measurements of the average characteristics. Moreover, by studying the holographic interferograms, the fluctuations in the coordinates of points of boundary layer separation and the size of the circulation zone can be determined with high accuracy, which yields a more complete representation of the flow around bodies.

NOTATION

T, temperature; n, water refractive index; α, heat elimination coefficient; q_t, thermal flux; t_w, temperature of the wall outer surface; f_{in}, temperature of the inner surface of the ebonite cylinder; Nu, Nusselt number; Re = Ud/ν, Reynolds number; U, free-stream velocity; d, cylinder diameter; D, channel width; λ_w, wall heat conductivity; $\Theta°$, angle measured downstream from the cylinder frontal point; ℓ, boundary layer thickness; L, wall thickness of the outer cylinder; n, distance from the cylinder surface along its normal.

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COMPUTATIONAL AND EXPERIMENTAL INVESTIGATION OF NONSTATIONARY HEAT TRANSFER AT A CRITICAL POINT

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UDC 536.244

Similarity criteria are derived for the conjugate problem of nonstationary heat transfer, and the behavior of the heat transfer coefficient with a sharp change in the boundary conditions is studied.

Heat transfer in modern aircraft and rocket motors often occurs under nonstationary conditions. At the present time the accuracy with which the nonstationary temperature and thermal-stress fields in structures is determined is limited primarily by the accuracy of the boundary conditions, in particular, in the form of nonstationary heat-transfer coefficients. The fact that the heat-transfer coefficient is time-dependent when the conditions of heat transfer change sharply was confirmed experimentally in [1, 2]. In [3] similarity criteria characterizing the rate of change of the temperature of the wall were introduced and criterional dependences making it possible to calculate the nonstationary heat-transfer coefficient were derived. It follows from the estimates of [4] that for bodies in a gas flow the period of time during which the nonstationary heat-transfer coefficient differs from the corresponding stationary value under conditions of large Reynolds numbers Re > 10^4-10^5 does not exceed hundredths of a second. In [5, 6], however, it was found that this time is equal to 3-5 sec, the maximum excess is a factor of 2-3, and neglecting the time-dependence of the heat-transfer coefficient leads to significant errors in the calculation of the temperature of the wall. The present investigation was performed in order to determine more accurately the time dependence of the heat-transfer coefficient accompanying a change in the conditions of heat transfer.

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We shall study a flow in a neighborhood of the bow critical point of a blunt body. We introduce the velocities u_1 and $u_{1\infty}$, which are related with the longitudinal component of the velocity in the boundary layer and the external flow by the relations

$$u = xu_1(t,\ x,\ y);\ u_\infty = xu_{1\infty}(t,\ x). \tag{1}$$

We shall use the following dimensionless variables:

$$u_{1D} = u_{1d}/u_{1\infty};\ y_D = y_d \sqrt{\frac{\mu_\infty}{\rho_\infty u_{1\infty}}};\ v_D = v_d \sqrt{\frac{\mu_\infty}{\rho_\infty} u_{1\infty}};\ \mu_D = \mu_d \mu_\infty;$$

$$t_D = t_d/u_{1\infty};\ T_D = T_d T_{\infty 0};\ P_D = P_d \rho_{\infty 0} u_{1\infty}^2 D^2;\ \rho_D = \rho_d \rho_{\infty 0}.$$

In the indicated dimensionless variables in a neighborhood of the bow critical point at $x = 0$ the system of boundary-layer equations, describing the flat laminar nonstationary motion of a compressible gas, with the corresponding initial and boundary conditions as well as the coupling conditions can be written in the following form (the index d is dropped):

$$\frac{\partial \rho}{\partial t} + \rho u_1 + \frac{\partial}{\partial y} \rho v = 0;$$

$$\rho \frac{\partial u_1}{\partial t} + \rho v \frac{\partial u_1}{\partial y} = \frac{\partial}{\partial y} \mu \frac{\partial u_1}{\partial y} - \rho u_1^2 + \rho_\infty \frac{\partial u_{1\infty}}{\partial t} + \rho_\infty u_{1\infty}^2; \tag{2}$$

$$\rho \frac{\partial T}{\partial t} + \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \frac{\mu}{\Pr} \frac{\partial T}{\partial y} + (j-1) M_0^2 \frac{\partial P}{\partial t};$$

$$P = \frac{1}{jM_0^2} \rho T;$$

$$T(0,\ y) = T_0(y);\ u(0,\ y) = u_0;$$

$$y \to \infty,\ T = T_\infty(t);\ P = P_\infty(t);\ u = u_\infty(t); \tag{3}$$

$$y = 0,\ \frac{\partial T_w}{\partial y} = 0; \tag{4}$$

$$y = \Delta \sqrt{\frac{\rho_\infty u_{1\infty}}{\mu_\infty}};\quad T_w = T;\ \lambda_w \frac{\partial T_w}{\partial y} = \frac{\partial T}{\partial y};\ u = v = 0; \tag{5}$$

$$0 < y < \Delta \sqrt{\frac{\rho_\infty u_{1\infty}}{\mu_\infty}};\ (\rho c)_w \frac{\partial T_w}{\partial t} = \frac{1}{\Pr} \frac{\partial}{\partial y} \lambda_w \frac{\partial T_w}{\partial y}. \tag{6}$$

We shall study the important practical case of a thin wall, when the temperature drop in the wall is negligibly small. Integrating Eq. (6) using Eq. (5) we obtain

$$\frac{\partial T}{\partial t} = \frac{1}{K} \frac{\partial T}{\partial y};\ K = (\rho c)_w \Delta \sqrt{\frac{\rho_\infty u_{1\infty}}{\mu_\infty}} \Pr. \tag{7}$$

If the body in the flow is a circular cylinder with diameter D, then, as follows from the theory of potential flow around a body (for example, [7]),

$$u_{1\infty} = 4U_\infty/D \text{ and } K = 2(\rho c)_w \frac{\Delta}{D} \sqrt{\mathrm{Re}} \Pr;\ \mathrm{Re} = \frac{\rho_\infty U_\infty D}{\mu_\infty}.$$

Let us assume that the flow is made nonstationary by a sharp change in the conditions of the external flow around the body (3) or the temperature of the wall. In this case both the degree of coupling of the problem and the degree of nonstationariness will be determined by the parameter K. As $K \to \infty$ the problem becomes uncoupled, and the time of the nonstationary process is determined by the rate at which disturbances build up in the boundary layer; for $K \sim 1$ the nonstationary nature of the problem is related with the rate of change of the temperature of the wall. The similarity parameter K depends on the physical properties of the body and the medium flowing around it, the thickness of the wall, the radius of curvature, and Reynolds number. For low flow velocities, as shown in [2], the heat transfer coefficient is different from its stationary value throughout the entire nonstationary process. Under otherwise equal conditions, owing to the difference in the densities the similarity parameter K for flow of water around bodies ($\rho \approx 1000$ kg/m^3) will be approximately three orders of magnitude smaller than in the case of flow of air ($\rho \approx 1.2$ kg/m^3), so that in the case of a body in a water flow heat transfer is stabilized more slowly.

The solution of the system of boundary-layer equations (2) with the corresponding initial and boundary conditions and the coupling condition (7) was found numerically with the help of an implicit difference scheme, described in detail in [8]. With the help of the transformation of variables $\tau = t$ and $\eta = y/\delta$ the equations can be written in the coordinates of the boundary layer in the standard manner [$P(t) = \text{const}$]:

$$\frac{\partial}{\partial\eta}\Gamma_\Phi\frac{\partial\Phi}{\partial\eta} - D_\Phi\frac{\partial\Phi}{\partial\eta} = \frac{\delta^2}{T}\frac{\partial\Phi}{\partial\tau} + S_\Phi;\quad \frac{\partial v}{\partial\eta} = S_v;$$
$$\Phi = T,\ u_1;\ \Gamma_u = \mu;\ \Gamma_T = \frac{\mu}{\mathrm{Pr}};\ D_u = D_T = \frac{v\delta}{T} - \eta\delta\frac{\partial\delta}{\partial\tau};\tag{8}$$
$$S_T = 0;\quad S_{u_1} = \delta^2\left(\frac{u_1^2}{T} - \frac{1}{T_\infty}\frac{\partial u_{1\infty}}{\partial\tau} - \frac{u_{1\infty}^2}{T_\infty}\right);$$
$$S_v = -\frac{\delta u_1}{T} + \frac{\delta}{T^2}\frac{\partial T}{\partial\tau} - \frac{\eta}{T^2}\frac{\partial\delta}{\partial\tau}\frac{\partial T}{\partial\eta}.$$

The convective and diffusion terms are written in the finite-difference form with the help of the hybrid notation [9] and the time derivatives are written in the standard manner:

$$\frac{\partial\Phi}{\partial\tau} = \frac{\Phi_i - \Phi_{i-1}}{\Delta\tau},$$

where i enumerates the time step. In practice the calculation is performed as follows. Assume that the solution is obtained at the i-th time step; this solution is used as the initial distribution for carrying out the iteration cycle, consisting of successive calculation of the system of equations (8). The first two equations are solved with the help of the difference factorization method. The problem is assumed to be solved, if for all points the condition of convergence of the iteration process

$$\left|1 - \frac{T_j^k}{T_j^{k-1}}\right| < \varepsilon,\quad 0 \leqslant j \leqslant N,\tag{9}$$

where k is the number of the iteration, is satisfied. After two to three iterations are performed the condition is checked at the boundary of the boundary layer

$$|(T_N - T_{N-1})/(T_N - T_0)| < \varepsilon_1.\tag{10}$$

If this condition is not satisfied, then the thickness of the boundary layer is increased and the calculation is continued until the conditions (9) and (10) are satisfied. The time step was chosen so that the increment to the boundary layer would not be very large.

We shall study heat transfer between a thin-walled steel cylinder 0.04 m in diameter with a wall thickness of 0.001 m and with water flowing around it in the transverse direction. We shall assume that the thermophysical properties of the water do not depend on the temperature, and Pr = 5. To perform the calculations the equation of state must be eliminated from the system of equations (2) and we must set ρ = const. A test calculation of the velocity profiles with a steady flow on a nonuniform computational grid with 50 cells showed that with accuracy up to the third decimal point the solution obtained agrees with the exact solution of the boundary-layer equations [7]. Figure 1 shows the distribution of the relative increase of the Nuesselt number as a function of the running time of the nonstationary heating (cooling) process of the wall for different Reynolds numbers. One can see that initially the nonstationary Nusselt number is always much greater than its stationary value, and the period of time during which this excess is significant rapidly decreases as the Raynolds number increases. In practical calculations the heat-transfer coefficient can be regarded as constant and equal to its stationary value already for Re $>10^3$.

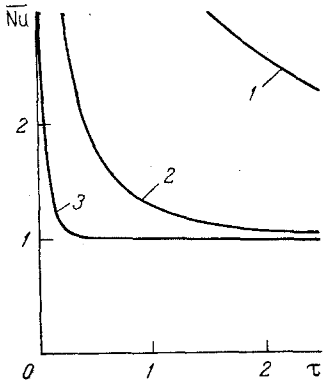


Fig. 1

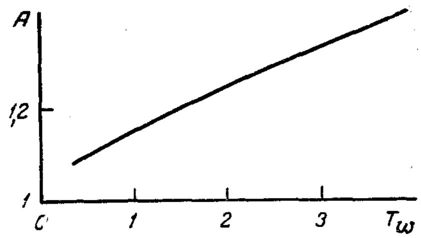


Fig. 2

Fig. 1. Relative increase in the Nusselt number $\overline{Nu}$ = Nu_{nons}/Nu_s versus the time τ, sec, for a cylinder in a water flow: 1) Re = 100; 2) 1000; 3) 10,000.

Fig. 2. Coefficient A in the formula $Nu = A\sqrt{Re}$ as a function of the temperature factor at the critical point for Pr = 1.

For air flowing around a cylinder of the same type the viscosity was approximated by Sutherland's formula

$$\mu/\mu_0 = (T/T_{\infty_0})^{3/2} \frac{T_{\infty_0} + C}{T + C}, \quad C = 110 \text{ K}.$$

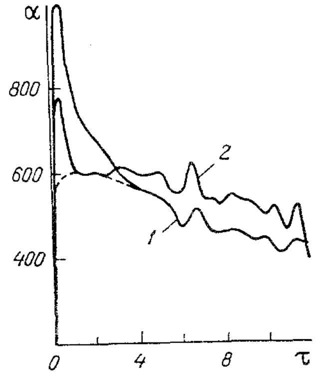
The dependence of the heat-transfer coefficient on the temperature factor under the conditions of steady flow and T_{∞_0} = 300 K is presented in Fig. 2. For T_w = 4 the heat-transfer coefficient is approximately 20% higher than its value at $T_w \simeq 1$. Because the density of air is low, heat transfer is stabilized more rapidly than in the case of the water flow. Thus, when the wall temperature increases abruptly the time period during which the nonstationary heat-transfer coefficient is at least 1.5 times greater than its stationary value is equal to 0.1 sec for Re = 10^2 and 0.002 sec for Re = 10^4.

Experimental investigations of the effect of the thermal nonstationariness on the heat-transfer coefficient were performed on a high-temperature stand in regimes in which the temperature dropped from T_g = 1200 K to T_g = 350 K. In the model studied — a thin-walled cylinder 0.04 m in diameter, 0.06 m long, and 0.001 m thick — a groove 0.0005 m deep and 0.012 m wide was made in the transverse section. Thus, in one experiment we were able to study the time dependence of the heat-transfer coefficient for two wall thicknesses — 0.001 and 0.0005 m. The temperature of the gas was measured with a two-junction chromel—alumel thermocouple with butt-welded junctions 0.0003 and 0.0005 m in diameter. The true gas temperature was determined by extrapolating the indications to zero diameter of the thermoelectrode. The heat-transfer coefficient was determined by an indirect method, based on measurement of the temperature of the inner thermally insulated surface of the wall [10]. The temperature of the inner surface of the model was measured at 18 points with chromel-alumel thermocouples 0.2 mm in diameter, whose hot junctions were first flattened and then point-welded to the inner surface of the model, and the heat-transfer coefficient was calculated using the formula

$$\alpha = -\frac{\lambda}{\Delta} \sum_{k=1}^{\infty} \frac{1}{(2k-1)!} \left(\frac{\rho c \Delta^2}{\lambda}\right)^k \frac{\partial^k T_w}{\partial \tau^k} \Big/ \left(T_w + \sum_{k=1}^{\infty} \frac{1}{(2k)!} \left(\frac{\rho c \Delta^2}{\lambda}\right)^k \frac{\partial^k T_w}{\partial \tau^k} - T_g\right).$$

As is well known, for Bi < 0.1 the heat-transfer coefficient can be calculated using only the first term in the expansion (the so-called thin-body method):

$$\alpha = -\rho c \Delta \frac{\partial T_w}{\partial \tau} \Big/ (T_w - T_g).$$



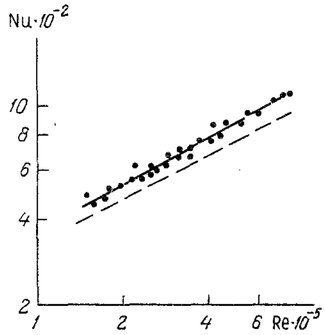


Fig. 3

Fig. 4

Fig. 3. Heat-transfer coefficient α [in W/(m²·deg)] as a function of time τ (in sec) of the nonstationary process with $Re = 4 \cdot 10^5$: the measurements were performed with a wall thickness of 0.0005 m (1) and 0.001 m (2); the dashed line corresponds to a calculation performed with the gas temperature shifted relative to the measured value by 0.3 sec.

Fig. 4. Nusselt's number versus Reynolds number: the dots are the experimental points and the dashed line shows the theoretical dependence $Nu = 1.04\sqrt{Re}$.

The method presented above has the drawback that is impossible to determine accurately T_h and α initially, when the first derivatives of $T_h(Fo)$ are close to zero, while the higher order derivatives are large and make a significant contribution to the sum in spite of the low values of the coefficients multiplying them. Estimates showed that for the numbers Bi ~ 0.01-0.05 obtained in the work the critical dimensionless time Fo*, prior to which the solution cannot be reconstructed, is Fo* ~ 0.5 or in physical units τ* ~ 0.12 sec.

The use of an automated system for acquiring and processing the experimental data made it possible to increase substantially the accuracy of the secondary processing. The indications of the thermocouples were amplified by a block of amplifiers and recorded on a 12-channel tape recorder. The experimental results were processed after the experiment ended and the processing included a number of stages: writing of the primary experimental data in a file in computer memory, digital filtering of the signal, secondary processing, and graphing of the results obtained. All programs, except for the secondary-processing program, were standard. Figure 3 shows the typical time dependences of the heat-transfer coefficient, obtained at the critical point. One can see that the heat-transfer coefficient increases sharply in the first second of the nonstationary process; this is explained by the finite time constant of the air thermocouple. If the results on the gas temperature, shifted relative to the measured temperature by 0.3 sec, are analyzed, then the sharp overshoot in the dependence of the heat-transfer coefficient vanishes. The general tendency for the heat-transfer coefficient to decrease with time can be explained by the effect of the temperature factor. This is explained by the fact that the heat-transfer coefficient measured on a thin wall (Δ = 0.0005 m) is lower than the analogous coefficient measured on a thicker wall (Δ = 0.001 m). Figure 4 shows the dependence of the Nusselt number, measured with a temperature factor close to unity, on the Reynolds number. The data are in good agreement with the theoretical dependence $Nu = 1.04\sqrt{Re}$.

In [5] it was found in a study of heat transfer on the bow arc of a circular cylinder with an angle of 120° that for analogous values of the coefficient of nonstationariness $K^* = \frac{1}{T_w}\frac{\partial T_w}{\partial \tau}\sqrt{\frac{\lambda}{c_p g \rho u}}$ the nonstationary heat-transfer coefficient is 40 to 60% higher than its stationary value. Apparently, because of the small geometric dimensions of the model (D = 0.006 mm, Δ = 0.001 mm) the flow of heat was significant. Initially, when the difference of the temperatures between the air and the wall was much greater than the difference of the temperatures of the cylinder wall, the heat-transfer coefficient at the critical point could be calculated from the indications of a thermocouple placed at the critical point of the cylinder. As the model cools (or is heated) the contribution of heat flow to the nonstationary process increases and some average heat-transfer coefficient over the arc of a circle, lower than the heat-transfer coefficient at the critical point, can be reconstructed from the indications of the thermocouple placed at the critical point. This is why a stationary dependence was obtained in [5] for the Nusselt number averaged over the arc of a circle $Nu = 0.635\sqrt{Re}$.

NOTATION

x and y, axes of a Cartesian coordinate system; t, time; u and v, longitudinal and transverse components of the velocity; ρ, density; P, pressure; T, temperature; μ, coefficient of viscosity; D, diameter of the cylinder; Δ, thickness of the wall; λ, thermal conductivity; c_p, heat capacity at constant pressure; g, acceleration of gravity; Re, Reynolds number; Pr, Prandtl's number; Nu, Nusselt's number; Bi, Biot's number; Fo, Fourier's number. Indices: ∞, potential flow; 0, initial conditions; w, wall; g, gas.

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